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Towards the First Compelling Signs of **Dynamical Dark Energy** *in the Expanding Universe*

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Guidelines of the Talk

- 99 years of the CC term in Einstein's equations!
- Vacuum energy and the CC Problem
- Inflation and vacuum decay in the early universe
- Running Vacuum Models
- RVM's versus observations: challenging the LCDM
- Conclusions

➢ Einstein's Equations

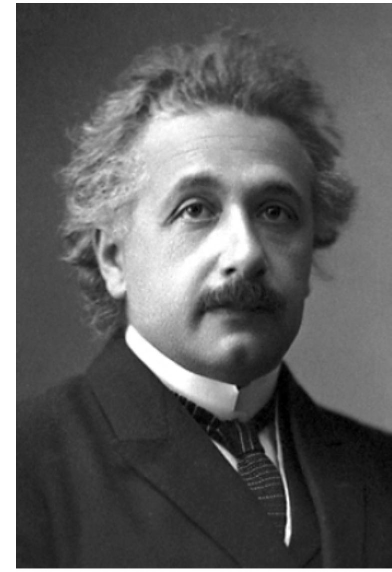
844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

Der Entwicklungsgang war dabei folgender. Zunächst fand ich Gleichungen, welche die Newtonsche Theorie als Näherung enthalten



1915 – 1916

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R = 8\pi G\, T_{\mu\nu}$$

A. Einstein, *Grundgedanken der allgemeinen Relativitätstheorie und Anwendung diese Theorie in der Astronomie*, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin (1915) 315; *Zur allgemeinen Relativitätstheorie*, Sitzungsber. Preuss. Akad. Wiss. Berlin (1915) 778; *Zur allgemeinen Relativitätstheorie (Nachtrag)*, Sitzungsber. Preuss. Akad. Wiss. Berlin (1915) 799; *Die Feldgleichungen der Gravitation*, Sitzungsber. Preuss. Akad. Wiss. Berlin (1915) 844; *Die Grundlage der allgemeinen Relativitätstheorie*, Annalen der Physik **49** (1916) 770.

➢ Modified Einstein's Equations

1917 Λ appears

A. Einstein, *Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie*, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin, phys.-math. Klasse VI (1917) 142-152.

$$\mathbf{R}_{\mu\nu} - \frac{1}{2}\mathbf{g}_{\mu\nu}\mathbf{R} - \Lambda\,\mathbf{g}_{\mu\nu} = 8\pi\mathbf{G}_N\,\mathbf{T}_{\mu\nu}$$

Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie.

Von A. Einstein.

Ausgegeben am 15. Februar.

§ 4. Über ein an den Feldgleichungen der Gravitation anzubringendes Zusatzglied.

Die von mir vorgeschlagenen Feldgleichungen der Gravitation lauten für ein beliebig gewähltes Koordinatensystem

$$\left.\begin{aligned} G_{\mu\nu} &= -\varkappa\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right) \\ G_{\mu\nu} &= -\frac{\partial}{\partial x_\alpha}\begin{Bmatrix}\mu\nu \\ \alpha\end{Bmatrix} + \begin{Bmatrix}\mu\alpha \\ \beta\end{Bmatrix}\begin{Bmatrix}\nu\beta \\ \alpha\end{Bmatrix} \\ &\quad + \frac{\partial^2 \lg \sqrt{-g}}{\partial x_\mu \partial x_\nu} - \begin{Bmatrix}\mu\nu \\ \alpha\end{Bmatrix}\frac{\partial \lg \sqrt{-g}}{\partial x_\alpha} \end{aligned}\right\}. \qquad (13)$$

kommen analog ist. Wir können nämlich auf der linken Seite der Feldgleichung (13) den mit einer vorläufig unbekannten universellen Konstante $-\lambda$ multiplizierten Fundamentaltensor $g_{\mu\nu}$ hinzufügen, ohne daß dadurch die allgemeine Kovarianz zerstört wird; wir setzen an die Stelle der Feldgleichung (13)

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\varkappa\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right). \qquad (13a)$$

➢ Einstein's Universe in modern notation

- Until about 1930 almost everybody ``knew'' that the universe was static, in spite of the two important papers by Friedmann in 1922 and 1924 and Lemaitre's no less important work in 1927 (recall that Hubble's law is from 1929)

Condition for static Universe:

$$4\pi G \rho_m = \frac{1}{a^2} = \Lambda$$

Einstein himself accepted the idea of an expanding universe only much later

1931 Λ disappears $\rightarrow$ (Einstein's “blunder”?)

A. Einstein, *Zum kosmologischen Problem der allgemeinen Relativitätstheorie*, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin, phys.-math. Klasse, XII, (1931) 235.

G. Gamow, *My World Line, an Informal Autobiography* (The Viking Press, New York 1970)

The relevant paragraph says (I quote it from my own copy of this book, p. 44): “Much later, when I was discussing cosmological problems with Einstein, he remarked that the introduction of the cosmological term was the biggest blunder he ever made in his life. ”.

If we continue reading G. Gamow's autobiography, just after the sentence mentioned we find (I continue quoting from my own copy of this book, p. 44): “But this “blunder”, rejected by Einstein, is still sometimes used by cosmologists even today, and the cosmological constant denoted by the Greek letter Λ rear its ugly head again and again and again. ”

1932 Λ is **completely neglected** $\rightarrow$ (CDM model)

A. Einstein, W. de Sitter, *On the relation between the expansion and the mean density of the universe*, Proceedings of the National Academy of Sciences **18** (1932) 213214.

One can read there the following sentence : "Historically the term containing the "cosmological constant" λ [written lowercase in the text of the article] was introduced into the field equations in order to enable us to account theoretically for the existence of a finite mean density in a static universe. It now appears that in the dynamical case this end can be reached without the introduction of λ".

1934 Λ resuscitates !!

$$\rho_\Lambda = \frac{\Lambda}{8\,\pi\,G}$$

G. Lemaître discusses for the first time the idea of Λ as vacuum energy.

G. Lemaître, Proc. of the Nat. Acad. of Sci. **20** (1934) 12

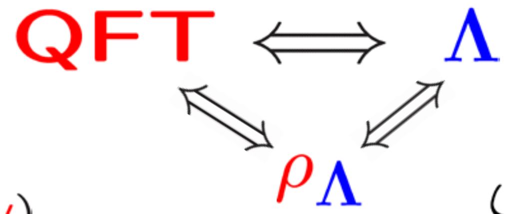
He had actually been discussing EE's with Λ since 1927...
independent of Friedmann first discussions
of dynamical solutions of EE's (1922)

2016…..Still with Λ and it will go on and on

Next year, 100 YEARS anniversary !!

Vacuum energy: zero-point energy and some cosmic numerology

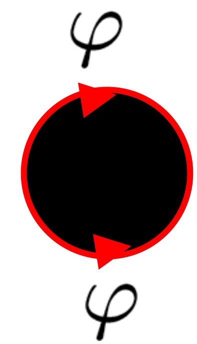
- Zeldovich (1967) first raised the connection



$$\rho_\Lambda = \frac{\Lambda}{8\,\pi\,G_N}$$ (vacuum energy density)

First thought: $\rho_\Lambda \propto m_p^4 \sim 1\mathrm{GeV}^4$

quantum bubble:



Impossible since $\rho_\Lambda \simeq \rho_c^0$ and $\rho_c^0 = \dfrac{3\,H_0^2}{8\pi\,G} \sim 10^{-47}\,\mathrm{GeV}^4$

Second thought:

$$\rho_\Lambda \simeq G\,m_p^6 = \frac{m_p^6}{M_P^2} \sim 10^{-38}\,\mathrm{GeV}^4$$

much better, but still unacceptable...

The CC problem (s)

Problem 1 : the "old" CC problem

Why all the big contributions to the DE add up to such a small value in Particle Physics units?

In the SM, $\Lambda_{\rm ph} = \Lambda_v + \Lambda_{SM}$ $\quad (\frac{\Lambda_{SM}}{\Lambda_{\rm ph}} \simeq \frac{10^8}{10^{-47}} \simeq 10^{55})$

Problem 2: the cosmic "coincidence" problem

Why the currently observed DE density is so close to the matter densisty?

coincidence ratio now: $r \equiv \frac{\rho_\Lambda^0}{\rho_M^0} = \frac{\Omega_\Lambda^0}{\Omega_M^0} \simeq \frac{7}{3} = \mathcal{O}(1)$

Λ in the SM and beyond

Source	Effect (GeV^4)	Λ/Λ_{exp}
electron 0-point	10^{-16}	10^{31}
QCD chiral	10^{-4}	10^{43}
QCD gluon	10^{-2}	10^{45}
Electroweak SM	10^{+9}	10^{56}
typical GUT	10^{+64}	10^{111}
Quantum Gravity	10^{+76}	10^{123} !!

!!

$$\rho_\Lambda^0 = \Omega_\Lambda^0 \, \rho_c^0 \simeq 6\, h^2 \times 10^{-47}\,\mathrm{GeV}^4 \simeq 3 \times 10^{-47}\,\mathrm{GeV}^4$$

$$m_\Lambda \equiv \sqrt[4]{\rho_\Lambda^0} \simeq 2 \ \mathrm{meV}$$

Λ in QFT: the Vacuum Energy

◇ Action integral for a scalar QFT:

$$S[\phi] = \int d^4x \sqrt{-g}\, \mathcal{L}(\phi, \partial_\mu \phi)$$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi\, \partial^\mu \phi - V_{eff}(\phi)$$

◇ Matter field energy-momentum tensor:

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = 2 \frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} - g_{\mu\nu} \mathcal{L}$$

$$= \left[\partial_\mu \phi\, \partial_\nu \phi - \frac{1}{2} g_{\mu\nu}\ \partial_\alpha \phi\, \partial^\alpha \phi \right] + g_{\mu\nu} V_{eff}$$

◇ For static equilibrium configurations $\Rightarrow$

$$< T_{\mu\nu} >= g_{\mu\nu}\ < V_{eff} >$$

generic **QFT** **problem** **!!**

➢ The old CC problem as a fine tuning problem

The CC problem stems from realizing that the effective or physical vacuum energy is the sum of two terms:

$$\rho_{\Lambda \text{phys}} = \rho_{\Lambda \text{vac}} + \rho_{\Lambda \text{ind}}$$

$$S_{EH} = \frac{1}{16\pi\, G_N} \int d^4x \sqrt{|g|}\, (R - 2\,\Lambda_{\text{vac}}) = \int d^4x \sqrt{|g|} \left(\frac{1}{16\pi\, G_N} R - \rho_{\Lambda \text{vac}} \right)$$

$$\rho_{\Lambda \text{vac}} = \frac{\Lambda}{8\pi\, G_N}$$

Vacuum bare term in Einstein eqs.

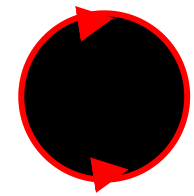
$$R_{ab} - \frac{1}{2} g_{ab} R = -8\pi\, G_N \left(\langle \tilde{T}^{\varphi}_{ab} \rangle + T_{ab} \right) = -8\pi\, G_N\, g_{ab}\, (\rho_{\Lambda \text{vac}} + \rho_{\Lambda \text{ind}} + T_{ab})$$

Quantum effects $\Rightarrow$ $\rho_{\Lambda \text{ind}} = \langle V(\varphi) \rangle + \text{ZPE}$

Zero-point energy in quantum field theory in flat spacetime

JS, arXiv:1306.1527

$$V_{\rm ZPE}(P) = \hbar\, V_P^{(1)} + \hbar^2\, V_P^{(2)} + \hbar^3\, V_P^{(3)} + \ldots.$$



Real scalar field, one loop:

$$V_P^{(1)} = (1/2)\sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} = (1/2)\sum_{\mathbf{k}} \hbar\sqrt{\mathbf{k}^2+m^2} \;\to$$

$$= \frac{1}{2}\int \frac{{\rm d}^3k}{(2\pi)^3}\sqrt{\mathbf{k}^2+m^2} = \frac{1}{4\pi^2}\int_0^{\Lambda_{\rm UV}} {\rm d}k\, k^2\sqrt{k^2+m^2}$$

$$= \frac{\Lambda_{\rm UV}^4}{16\pi^2}\left(1+\frac{m^2}{\Lambda_{\rm UV}^2}-\frac{1}{4}\frac{m^4}{\Lambda_{\rm UV}^4}\ln\frac{\Lambda_{\rm UV}^2}{m^2}+\cdots\right),$$

renormalization

$$V_P^{(1)\rm renorm} = -\frac{m^4}{64\pi^2}\ln\frac{\mu^2}{m^2}+\cdots$$

- In dimensional regularization:

$$V_P^{(1)} = \frac{1}{2}\,\mu^{4-n}\int \frac{d^{n-1}k}{(2\pi)^{n-1}}\,\sqrt{\mathbf{k}^2+m^2} = \frac{1}{2}\,\beta_\Lambda^{(1)}\,\left(-\frac{2}{4-n} - \ln\frac{4\pi\mu^2}{m^2} + \gamma_E - \frac{3}{2}\right)$$

with

$$\beta_\Lambda^{(1)} = \frac{m^4}{2\,(4\pi)^2} \qquad (\beta\text{-function coeff. for running } \Lambda)$$

How to get rid of the UV-part? Recall that

$$S_{\rm EH} = \frac{-1}{16\pi\,G^{(\rm b)}}\int d^4x\sqrt{-g}\,\left(R + 2\,\Lambda^{(\rm b)}\right) = -\int d^4x\sqrt{-g}\,\left(\frac{1}{16\pi\,G^{(\rm b)}}\,R + \rho_\Lambda^{(\rm b)}\right)$$

Next we split $\rho_\Lambda^{(\rm b)}$ into $\rho_\Lambda^{(\rm b)} = \rho_\Lambda(\mu) + \delta\rho_\Lambda$

$$\delta\rho_\Lambda^{\overline{\rm MS}} = \frac{m^4\,\hbar}{4\,(4\pi)^2}\,\left(\frac{2}{4-n} + \ln 4\pi - \gamma_E\right)$$

We obtain: $$\rho_\Lambda^{(\rm b)} + V_{\rm ZPE}^{(\rm b)} = \rho_\Lambda(\mu) + V_{\rm ZPE}(\mu)$$

where

$$V_{\rm ZPE}^{(1)}(\mu) = \hbar\, V_P^{(1)} + \delta\rho_\Lambda^{\overline{\rm MS}} = \frac{m^4\,\hbar}{4\,(4\,\pi)^2}\left(\ln\frac{m^2}{\mu^2} - \frac{3}{2}\right)$$

$$\rho_{\rm vac}^{(1)} = \rho_\Lambda(\mu) + V_{\rm ZPE}^{(1)}(\mu) = \rho_\Lambda(\mu) + \frac{m^4\,\hbar}{4\,(4\,\pi)^2}\left(\ln\frac{m^2}{\mu^2} - \frac{3}{2}\right)$$

overall μ-independent (RG-invariance)

$$d\rho_{\rm vac}^{(1)}/d\ln\mu = 0 \quad \Rightarrow \quad \mu\frac{d\rho_\Lambda(\mu)}{d\mu} = \frac{\hbar\, m^4}{2\,(4\pi)^2} = \beta_\Lambda^{(1)}$$

We have found the expected result:

$$V_P^{(1)\rm renorm} = -\frac{m^4}{64\pi^2}\ln\frac{\mu^2}{m^2} + \cdots$$

➢ "Fine" details of the **Fine tuning problem** !!

A little bit more of QFT stuff...

JS, arXiv:1306.1527

Take a scalar QFT with effective potential

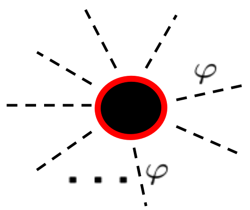
$$V_{\rm eff} = V + \hbar\, V_1 + \hbar^2\, V_2 + \hbar^3\, V_3 + ...$$

where

$$V_1 = V_P^{(1)} + V_{\rm scal}^{(1)}(\varphi)\,, \qquad V_2 = V_P^{(2)} + V_{\rm scal}^{(2)}(\varphi)\,, \qquad V_3 = V_P^{(3)} + V_{\rm scal}^{(3)}(\varphi)\ldots\,.$$

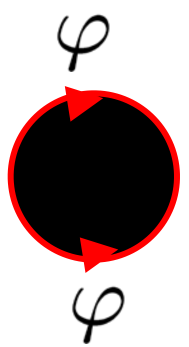
Thus,

$$\boxed{V_{\rm eff}(\varphi) = V_{\rm ZPE} + V_{\rm scal}(\varphi)}$$



with

$$V_{\rm ZPE} = \hbar\, V_P^{(1)} + \hbar^2\, V_P^{(2)} + \hbar^3\, V_P^{(3)} + \ldots.$$



➢ Recomputing the renormalized CC term in curved space-time

JS, arXiv:1306.1527

- *Adapting the formalism to a curved background*

Classical action now is:

$$S[\phi_c] = \int d^4x \sqrt{-g(x)} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi_c \partial_\nu \phi_c + \frac{1}{2} \xi \phi_c^2 R - V_c(\phi_c)\right]$$

For a free field $V_c(\phi_c) = (1/2)\, m^2 \phi_c^2$.

We restrict to this simpler case $\Rightarrow$ Even so, complicated! **!!**

The free field Green's function equation for the Feynman propagator in a curved background reads:

$$\left(\Box_x + m^2 - \xi R\right) G_F(x,x') = -\frac{\delta(x-x')}{\sqrt{-g(x)}} \qquad \text{with } \Box = g^{\mu\nu}\nabla_\mu\nabla_\nu.$$

- In the free field case in flat spacetime we had:

$$\left(\Box_x + m^2\right)\, G_F(x,x') = -\delta(x-x')$$

with the well-known solution

$$G_F(x,x') = \int \frac{d^4k}{(2\pi)^4}\,\frac{e^{ik(x-x')}}{k^2 - m^2 + i\epsilon}$$

- Need a generalization of this result to curved spacetime **!!**

EA is connected with quantum averaged $T_{\mu\nu}$: $\langle T^{\mu\nu}\rangle = -\frac{2}{\sqrt{-g}}\frac{\delta\Gamma_{\rm eff}}{\delta g_{\mu\nu}}$

Once more we have the 1-loop correction to the EA:

$$\Gamma^{(1)}_{\rm eff} = \frac{i\,\hbar}{2}\,Tr\,\mathcal{K}(x,x') = \frac{i\,\hbar}{2}\int d^4x\sqrt{-g}\,\lim_{x\to x'}\,\ln[\mathcal{K}(x,x')]$$

coincidence limit $x \to x'$

but here

$$\mathcal{K}(x,x') \equiv -G_F^{-1}(x,x') = \left(\Box_x + m^2 - i\epsilon - \xi\,R\right)\,\delta(x-x')\,\left[-g(x')\right]^{-1/2}$$

as confirmed by the fact that

$$\int d^4x'\sqrt{-g(x')}\,\mathcal{K}(x,x')\,G_F(x',x'') = -\frac{\delta(x-x'')}{\sqrt{-g(x'')}}$$

Quantum effects on the scalar action in **curved spacetime** generates

higher order geometric terms

which must be present in the classical action

in order to make it **renormalizable**:

$$\Gamma = S[\phi_c] + S_{\rm HD} + S_{\rm EH} + \Gamma_{\rm eff}^{(1)} = S[\phi_c] + S_{\rm HD}^{(1)} + S_{\rm EH}^{(1)}$$

$$S_{\rm HD}^{(1)} = S_{\rm HD} + \int {\rm d}^4x\sqrt{-g}\,\frac{\hbar}{2(4\pi)^2}\,\Delta(\mu)\,a_2(x)$$

$$= \int d^4x\sqrt{-g}\,\left(\alpha_1^{(1)}C^2 + \alpha_2^{(2)}\,R^2 + \ldots\right)$$

...and the **renormalized Hilbert-Einstein term:**

$$S_{\rm EH}^{(1)} = S_{\rm EH} + \int {\rm d}^4x\sqrt{-g}\;\frac{\hbar}{2(4\pi)^2}\,\Delta(\mu)\,\left(\frac{1}{2}\,m^4\,a_0 - m^2\,a_1(x)\right)$$

$$= \;\; -\int d^4x\sqrt{-g}\left(\frac{1}{16\pi G^{(1)}}R + \rho_{\rm vac}^{(1)}\right)$$

renormalized Newton's coupling

renormalized vacuum energy density

Fixing the counterterms:

$$\left\{\begin{array}{rcl} \alpha_1^{(1)} &=& \alpha_1(\mu) - \dfrac{\hbar}{2(4\pi)^2}\,\dfrac{1}{120}\left(\ln\dfrac{m^2}{\mu^2} + \text{finite const.}\right) \\ \alpha_2^{(1)} &=& \alpha_2(\mu) - \dfrac{\hbar}{4(4\pi)^2}\left(\dfrac{1}{6}-\xi\right)^2\left(\ln\dfrac{m^2}{\mu^2} + \text{finite const.}\right) \\ \dfrac{1}{16\pi G^{(1)}} &=& \dfrac{1}{16\pi G(\mu)} + \dfrac{\hbar\, m^2}{2(4\pi)^2}\left(\dfrac{1}{6}-\xi\right)\left(\ln\dfrac{m^2}{\mu^2} + \text{finite const.}\right) \\ \rho_{\rm vac}^{(1)} &=& \rho_\Lambda(\mu) + \dfrac{m^4\,\hbar}{4\,(4\,\pi)^2}\left(\ln\dfrac{m^2}{\mu^2} + \text{finite const.}\right)\,. \end{array}\right.$$

In particular, we've found the same expression for the 1-loop ρ_Λ as in flat spacetime !!

RGE's!!

$$\left\{\begin{array}{rcl} \dfrac{d\alpha_1(\mu)}{d\ln\mu} &=& -\dfrac{\hbar}{120(4\pi)^2} \equiv \beta_1^{(1)} \\ \dfrac{d\alpha_2(\mu)}{d\ln\mu} &=& -\dfrac{\hbar}{2(4\pi)^2}\left(\dfrac{1}{6}-\xi\right)^2 \equiv \beta_2^{(1)} \\ \dfrac{d}{d\ln\mu}\left(\dfrac{1}{16\pi G(\mu)}\right) &=& \dfrac{\hbar\, m^2}{(4\pi)^2}\left(\dfrac{1}{6}-\xi\right) \equiv \beta_{G^{-1}}^{(1)} \\ \dfrac{d\rho_\Lambda(\mu)}{d\ln\mu} &=& \dfrac{\hbar\, m^4}{2\,(4\pi)^2} \equiv \beta_\Lambda^{(1)}\,. \end{array}\right.$$

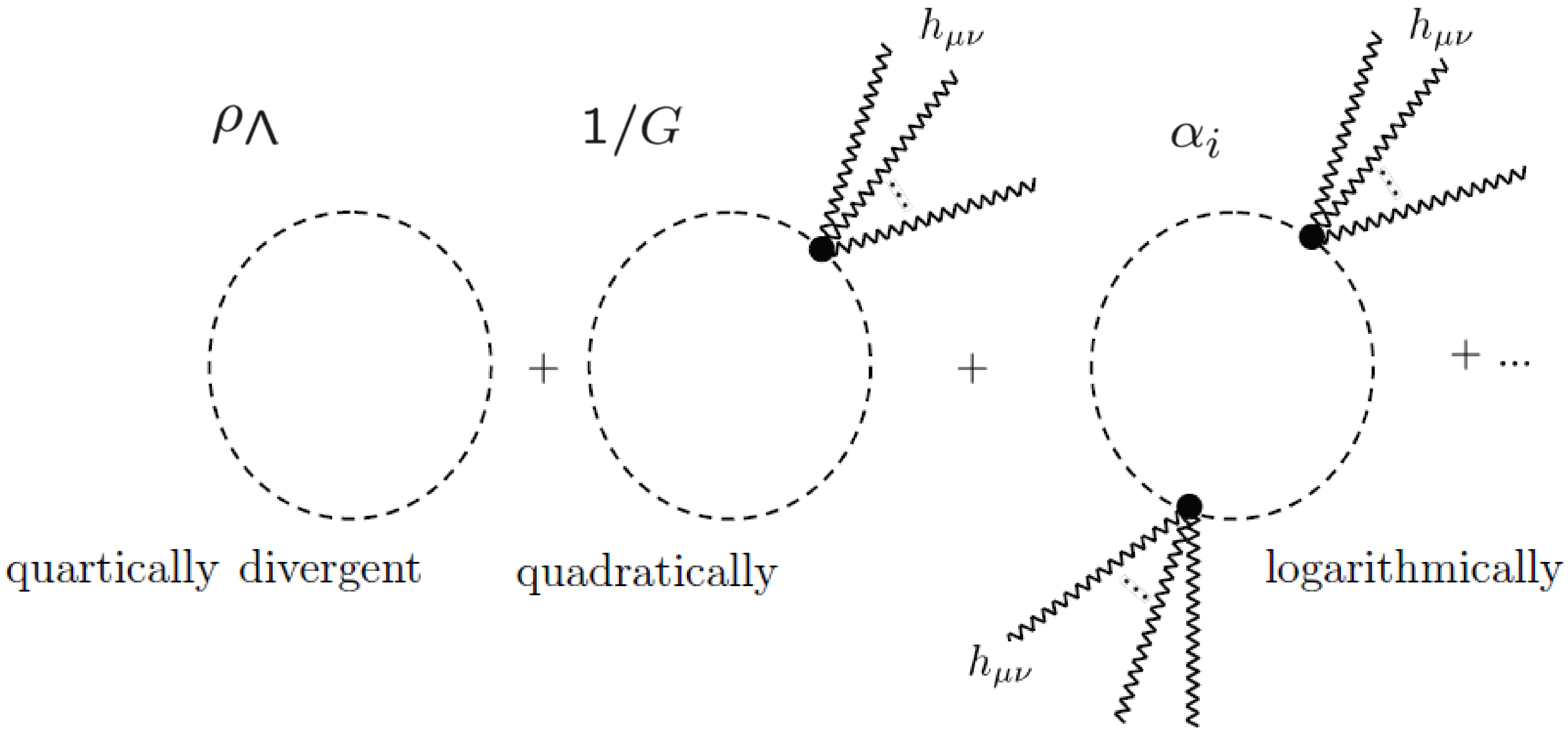
➢ Introducing an external gravitational field: QFT in curved spacetime!

In diagrammatic form, $\Rightarrow$ expansion $\sqrt{-g}$ around Minkowski space,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h = \eta^{\mu\nu} h_{\mu\nu}$$

$$\sqrt{-g} = 1 + \frac{1}{2}h + \frac{1}{8}h^2 - \frac{1}{4}h_{\mu\nu}h^{\mu\nu} + \mathcal{O}\left(h^3\right)$$



Think of a more physical interpretation of the running:

$$\frac{d}{d\ln\mu}\left(\frac{1}{16\pi G(\mu)}\right) = \frac{\hbar\, m^2}{(4\pi)^2}\left(\frac{1}{6}-\xi\right) \equiv \beta^{(1)}_{G^{-1}}$$

$\mu \to H$ (external tails)

$$G(H) = \frac{G_0}{1+\nu\,\ln\left(H^2/H_0^2\right)} \qquad G_0 \equiv G(H_0) = 1/M_P^2$$

with

$$\nu = \frac{1}{2\pi}\sum_i\left(\frac{1}{6}-\xi_i\right)\frac{m_i^2}{M_P^2} + \text{fermions}$$

Running G logarithmically and ρ_Λ quadratically

Basic set of equations:
(k=0)

$$G(H;\nu) = \frac{G_0}{1+\nu\ \ln\left(H^2/H_0^2\right)}$$

$$\rho + \rho_\Lambda = \frac{3H^2}{8\pi G}, \qquad \text{(Friedmann)}$$

$$(\rho + \rho_\Lambda)\, dG + G\, d\rho_\Lambda = 0 \qquad \text{(Bianchi)}$$

$$\rho_\Lambda = C_1 + C_2 H^2$$

$$C_2 = \frac{3\nu}{8\pi} M_P^2$$

$$\rho_\Lambda(H_0) = \rho_\Lambda^0 \quad \Longrightarrow \quad C_1 = \rho_\Lambda^0 - \frac{3\nu}{8\pi} M_P^2 H_0^2$$

More general running scenarios with G and Λ

Bianchi identity leads to $$\nabla^{\mu}\left[G\left(T_{\mu\nu}+g_{\mu\nu}\,\rho_{\Lambda}\right)\right]=0$$

$$\frac{d}{dt}\left[G(\rho+\rho_{\Lambda})\right]+3\,G\,H\,(\rho+p)=0\,.$$

Possible scenarios:

i) $G=$const. *and* $\rho_{\Lambda}=$const.$\Rightarrow \begin{cases}\dot{\rho}+3\,H\,(\rho+p)=0\\ \Lambda\text{CDM}\end{cases}$

ii) $G=$const *and* $\dot{\rho_{\Lambda}}\neq 0 \Rightarrow$ $\boxed{\dot{\rho_{\Lambda}}+\dot{\rho}+3\,H\,(\rho+p)=0}$

iii) $\dot{G}\neq 0$ *and* $\rho_{\Lambda}=$const. $\Rightarrow$

$$\dot{G}(\rho+\rho_{\Lambda})+G[\dot{\rho}+3H(\rho+p)]=0$$

iv) $\dot{G}\neq 0$ *and* $\dot{\rho_{\Lambda}}\neq 0 \Rightarrow$

$$\boxed{(\rho+\rho_{\Lambda})\dot{G}+G\dot{\rho_{\Lambda}}=0}$$

A semiclassical FLRW with running Λ

$$\rho_\Lambda = C_1 + C_2\, H^2\,.$$

I. Shapiro and J. Solà (2000,2003,2009)
J. Solà and H. Stefancic (2005,2006)
J. Solà (2007,2013)

Bianchi identity

$$\dot{\rho}_\Lambda + \dot{\rho}_m + 3\,H(\rho_m + p_m) = 0$$

(matter non-conservation!!)

$$C_2 \propto \nu = \frac{M^2}{12\pi\, M_P^2}$$

$$\rho_m(z) = \rho_m^0\,(1+z)^{3(1-\nu)}$$

and dynamical vacuum energy:

$$\rho_\Lambda(z) = \rho_\Lambda^0 + \frac{\nu\,\rho_m^0}{1-\nu}\left[(1+z)^{3(1-\nu)} - 1\right]$$

$\bar{\Lambda}$CDM: a vacuum model for the complete cosmic history

(RG) equation for the vacuum energy density of the expanding Universe

(reviews: J. Solà, arXiv:1306.1527; J. Solà, & A. Gómez-Valent, arXiv:1501.03832)

$$\frac{d\rho_\Lambda(\mu)}{d\ln\mu^2} = \frac{1}{(4\pi)^2}\left[\sum_i B_i M_i^2\,\mu^2 + \sum_i C_i\,\mu^4 + \sum_i \frac{D_i}{M_i^2}\,\mu^6 \;+ \ldots\right]$$

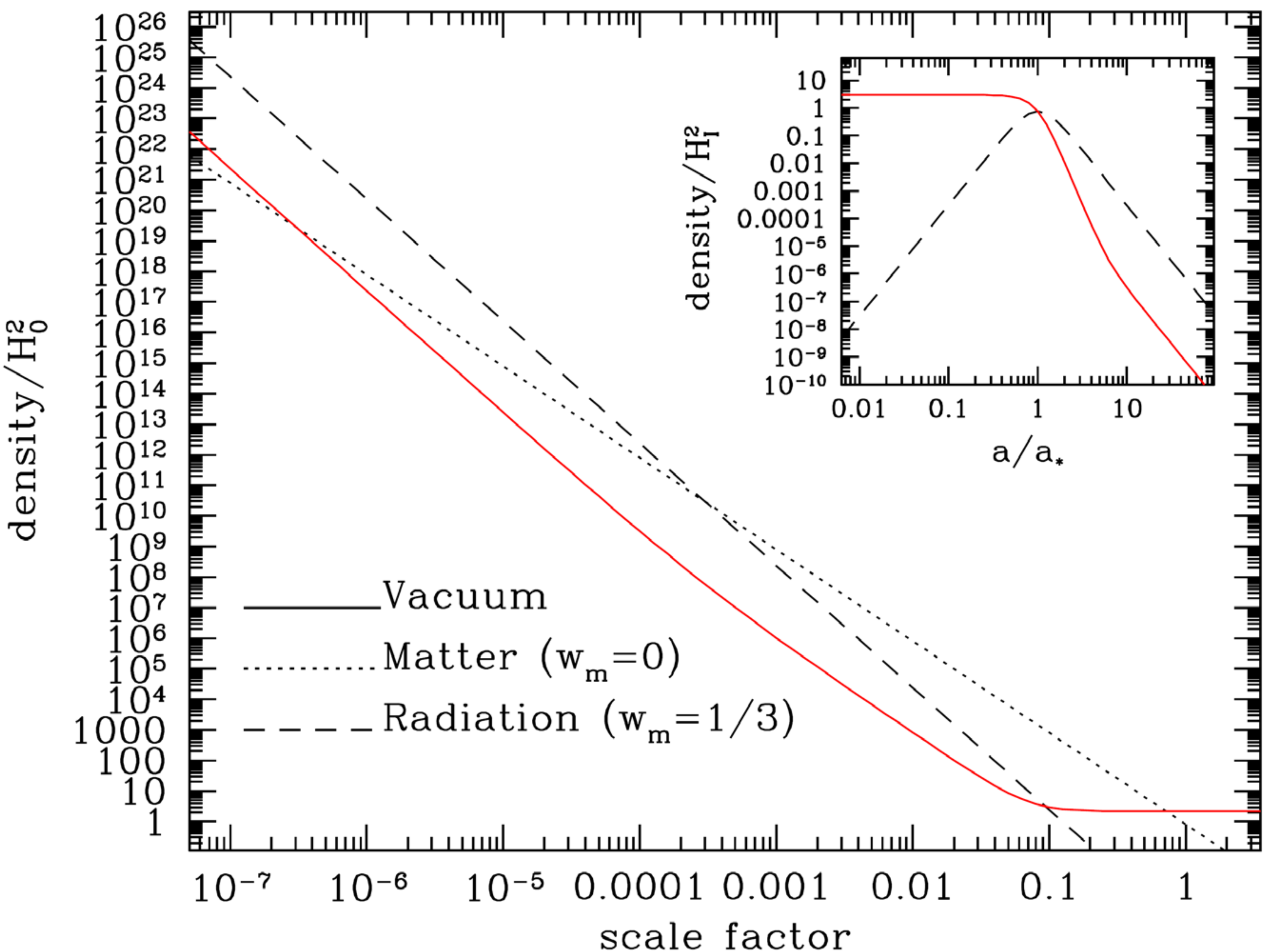
$\Downarrow$ $\mu^2 = aH^2 + b\dot{H}$

$$\rho_\Lambda(H,\dot{H}) = a_0 + a_1\,\dot{H} + a_2\,H^2 + a_3\,\dot{H}^2 + a_4\,H^4 + a_5\,\dot{H}\,H^2$$

$\Downarrow$ $\mu^2 = H^2$

$$\rho_\Lambda(H) = \frac{3}{8\pi G_N}\left(c_0 + \nu H^2 + \frac{H^4}{H_I^2}\right)$$

with S. Basilakos and J.A.S. Lima, arXiv:1209.2802 (MNRAS 2013)

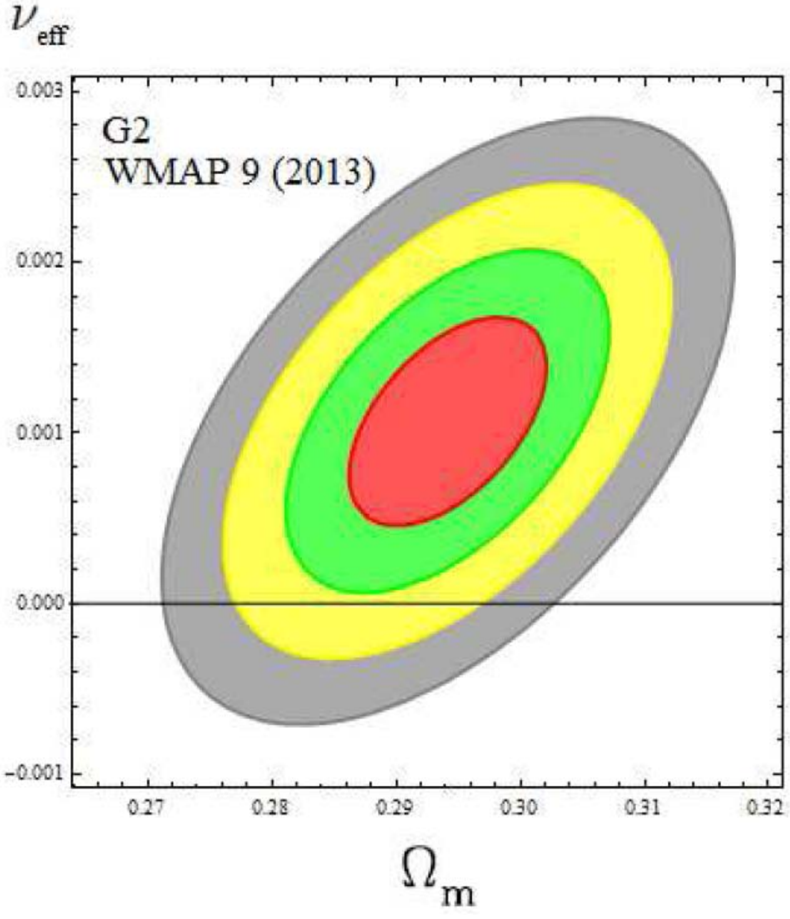


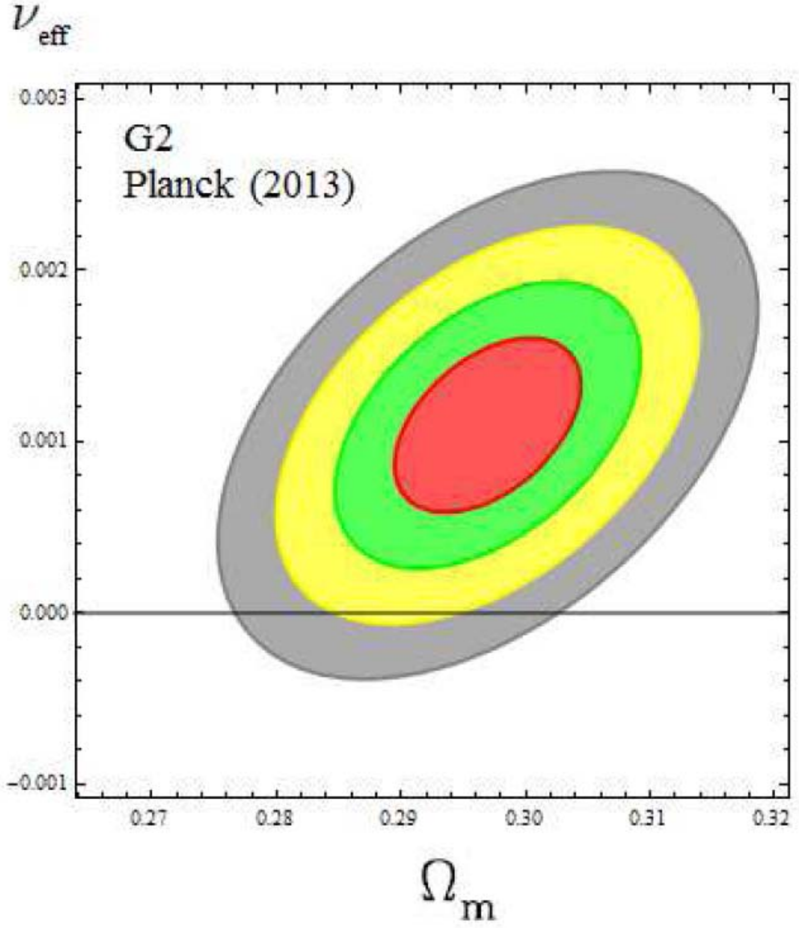
➢ First evidence of Running Cosmic Vacuum with fixed/variable G

with my students A. Gómez-Valent, J. de Cruz Pérez

arXiv:1506.05793 (ApJ Lett. 2015)

arXiv:1602.02103 (to appear in ApJ)

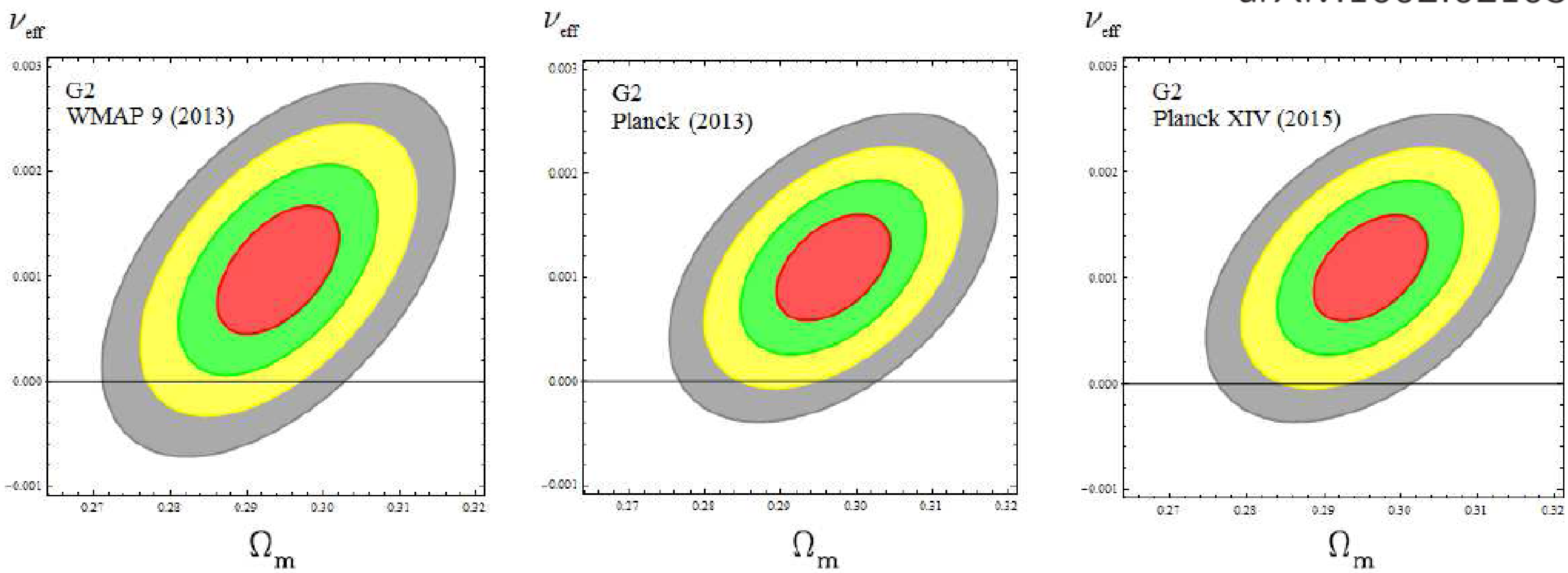




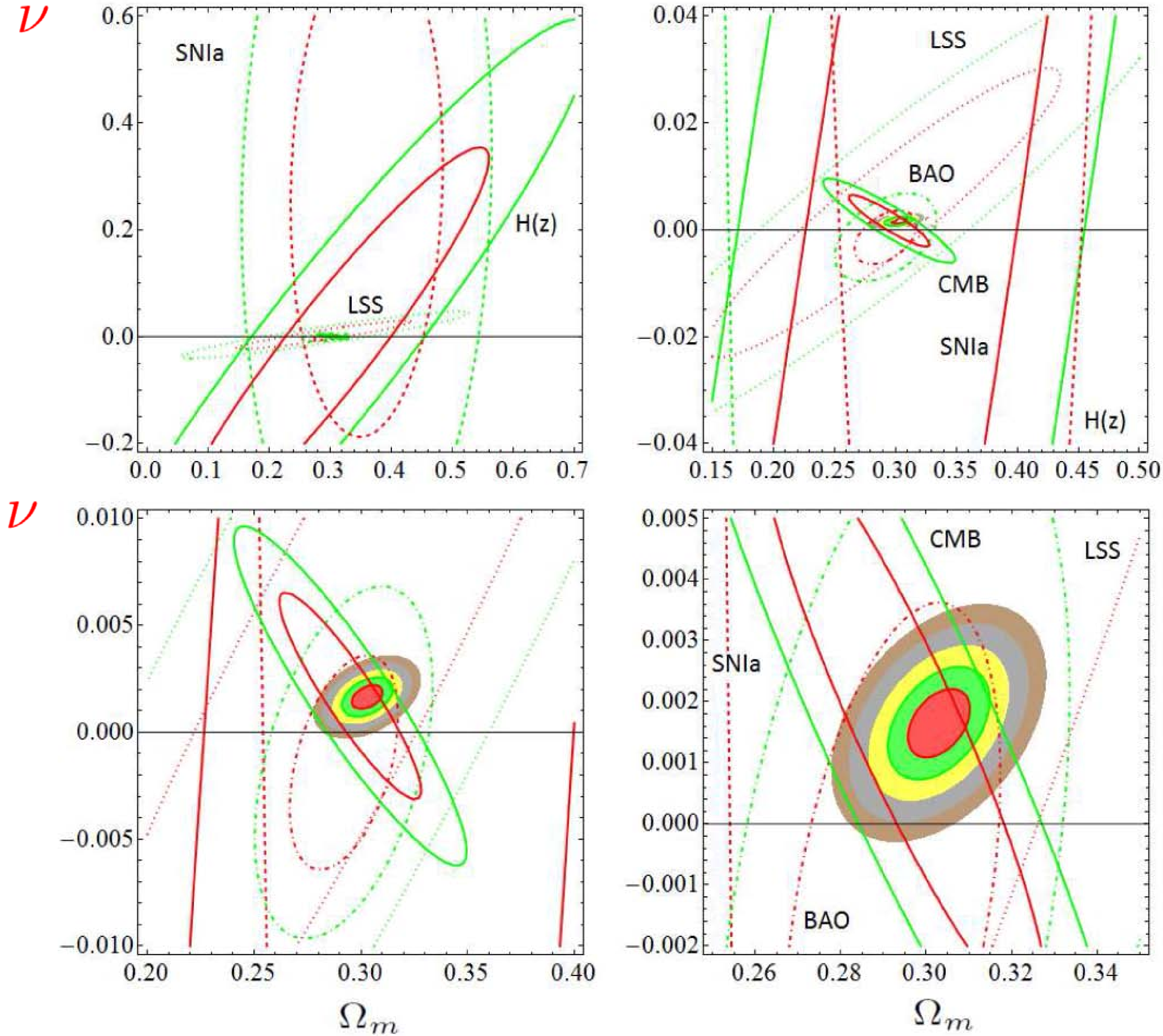
SNIa+BAO+$H(z)$+LSS+CMB data

- Increasing evidence in favor of of Running Cosmic Vacuum from WMAP to Planck 2013 and Planck 2015

arXiv:1602.02103



WMAP → Planck



ν
SNIa
H(z)
LSS
BAO
CMB
Ω_m

$$\rho_\Lambda(H) = \frac{3}{8\pi G}\left(c_0 + \nu H^2\right)$$

$$c_0 = H_0^2\left(\Omega_\Lambda - \nu\right)$$

Notice that the fact that $\nu > 0$

$\nu = 0.00152 \pm 0.00037$

4σ c.l. !!

RVM's behave as **effective quintessence**

➢ Dynamical Dark Energy as Quintessence versus Dynamical Vacuum

with my students A. Gómez-Valent and J. de Cruz Pérez (arXiv:1610.08965)

ϕCDM with Peebles & Ratra potential

$$\rho_\phi = \frac{M_{pl}^2}{16\pi}\left[\frac{\dot{\phi}^2}{2} + V(\phi)\right], \quad p_\phi = \frac{M_{pl}^2}{16\pi}\left[\frac{\dot{\phi}^2}{2} - V(\phi)\right]$$

$$\boxed{V(\phi) = \frac{1}{2}\kappa M_{pl}^2 \phi^{-\alpha}}$$

"tracker condition"

$$\Gamma \equiv V\,V''/(V')^2 > 1$$

$$\Gamma = 1 + 1/\alpha \;\; > 1$$

(for $\alpha > 0$)

same SNIa+BAO+$H(z)$+LSS+CMB **data**

5-dimensional fitting vector:

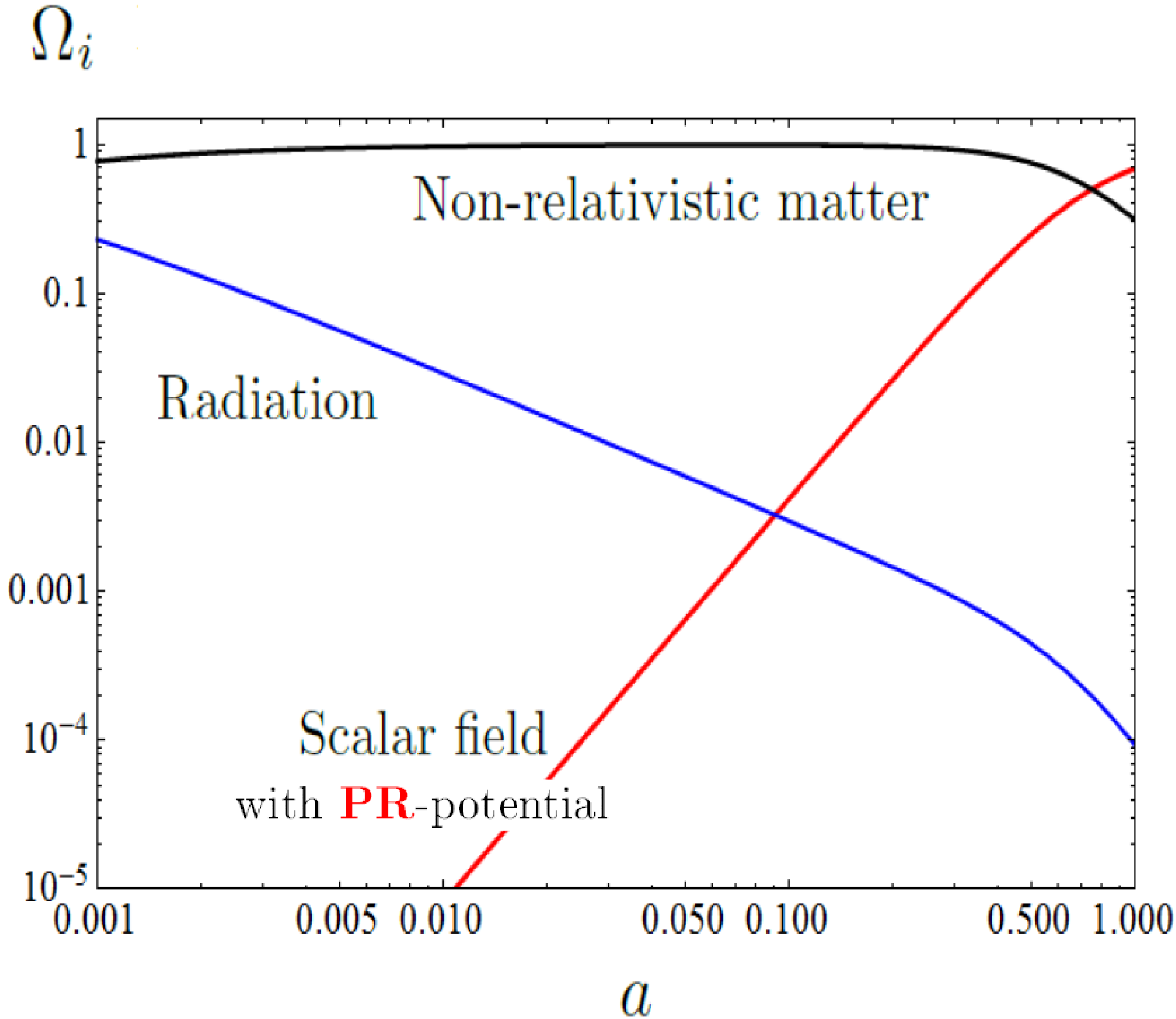
$$\mathbf{p}_{\phi\mathrm{CDM}} = (\omega_m, \omega_b, n_s, \alpha, \bar{\kappa})$$

$$\omega_m = \Omega_m h^2 \qquad \omega_b = \Omega_b h^2$$

$$\kappa\, M_P^2 \equiv \bar{\kappa}\,\varsigma^2 \qquad H_0 \equiv 100 h\,\varsigma$$

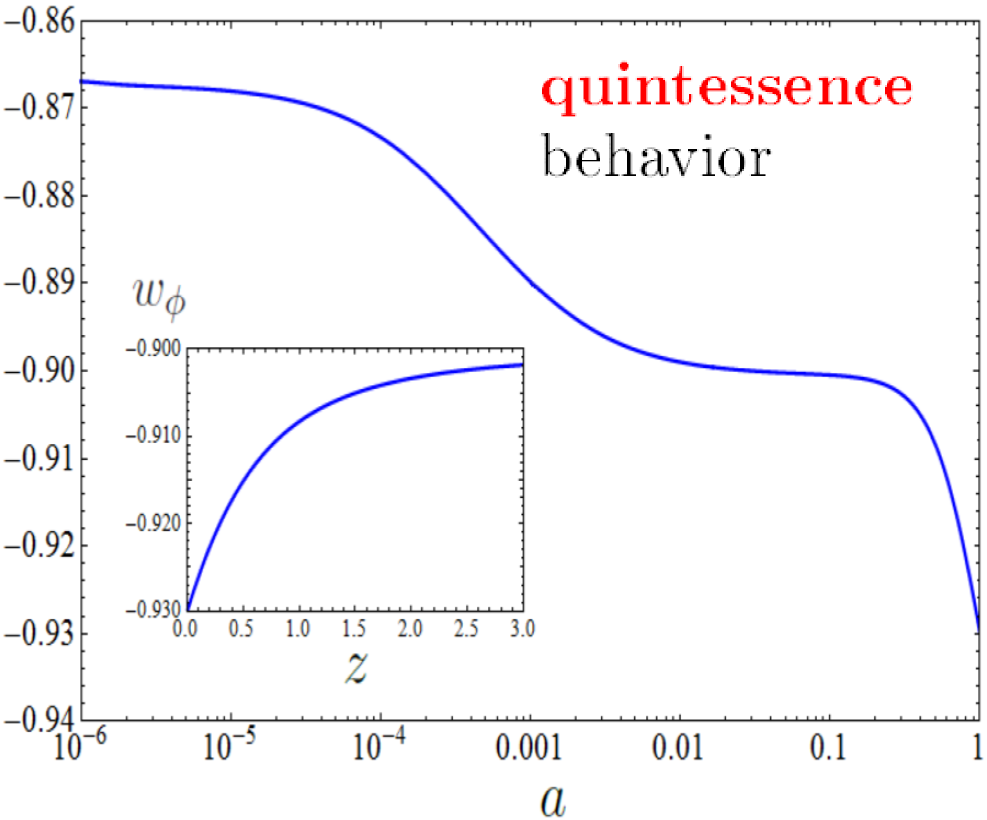
$$\varsigma \equiv 1 Km/s/Mpc$$

$$= \; 2.133 \times 10^{-44} GeV \;\; (\text{in natural units})$$



Ω_i
1
0.1
0.01
0.001
10^{-4}
10^{-5}
Non-relativistic matter
Radiation
Scalar field
with PR-potential
0.001
0.005
0.010
0.050
0.100
0.500
1.000
a

Exact **EoS** function



$$w_\phi = -0.931 \pm 0.017$$

at $z = 0$ → 4σ c.l. **!!**

initial conditions

$$\phi(t) = A\, t^p$$

$$\rho(a) = \rho_0 a^{-n}$$

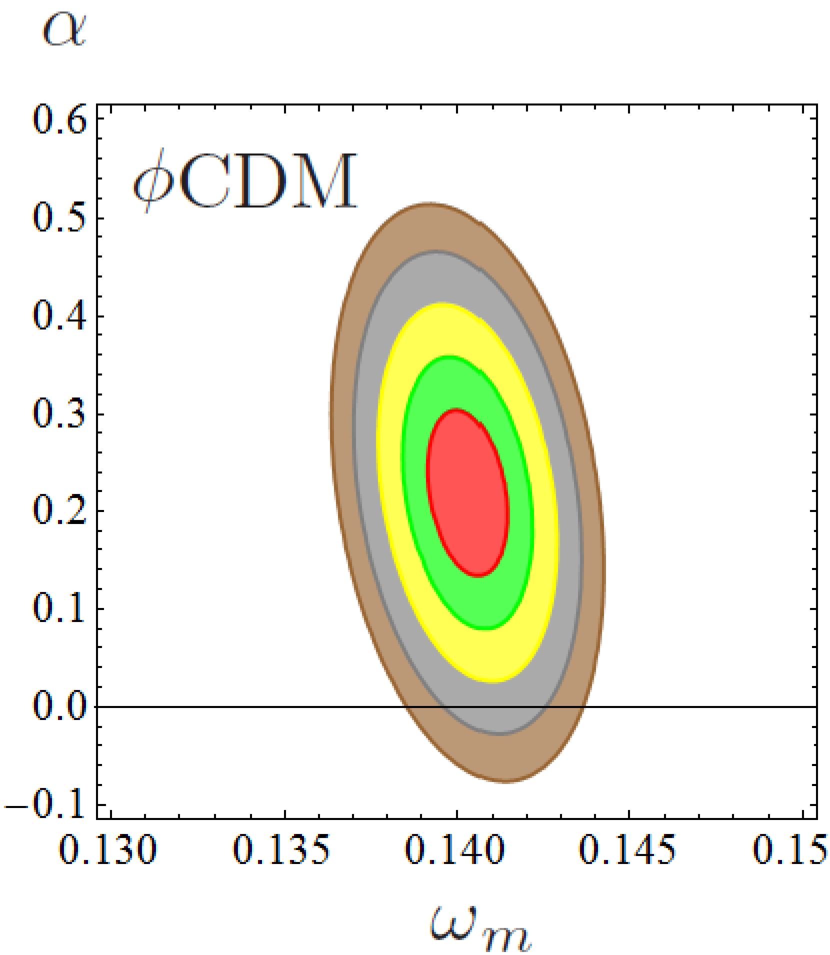
KG-equation with **PR**-potential ↓

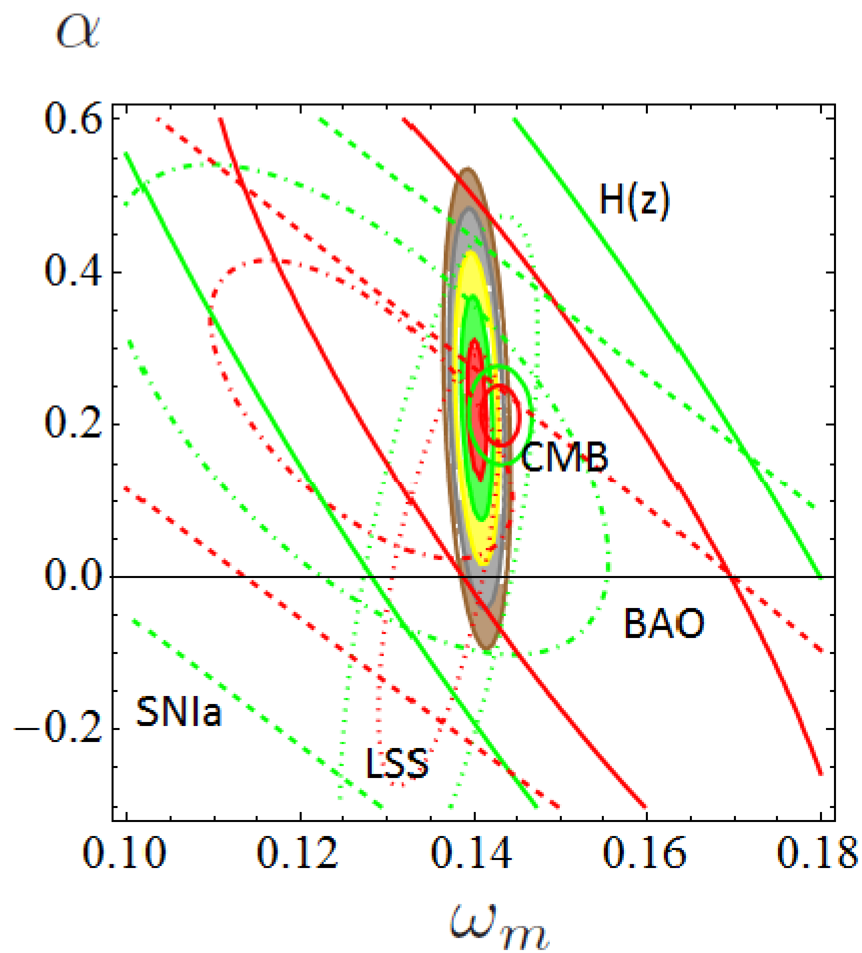
$$p = \frac{2}{\alpha + 2}$$

$$A^{\alpha+2} = \frac{\alpha(\alpha+2)^2 M_{pl}^2 \kappa n}{4(6\alpha + 12 - n\alpha)}$$

$$w_\phi = \frac{p_\phi}{\rho_\phi} = -1 + \frac{\alpha n}{3(2+\alpha)}$$

Contour Plots and their reconstruction

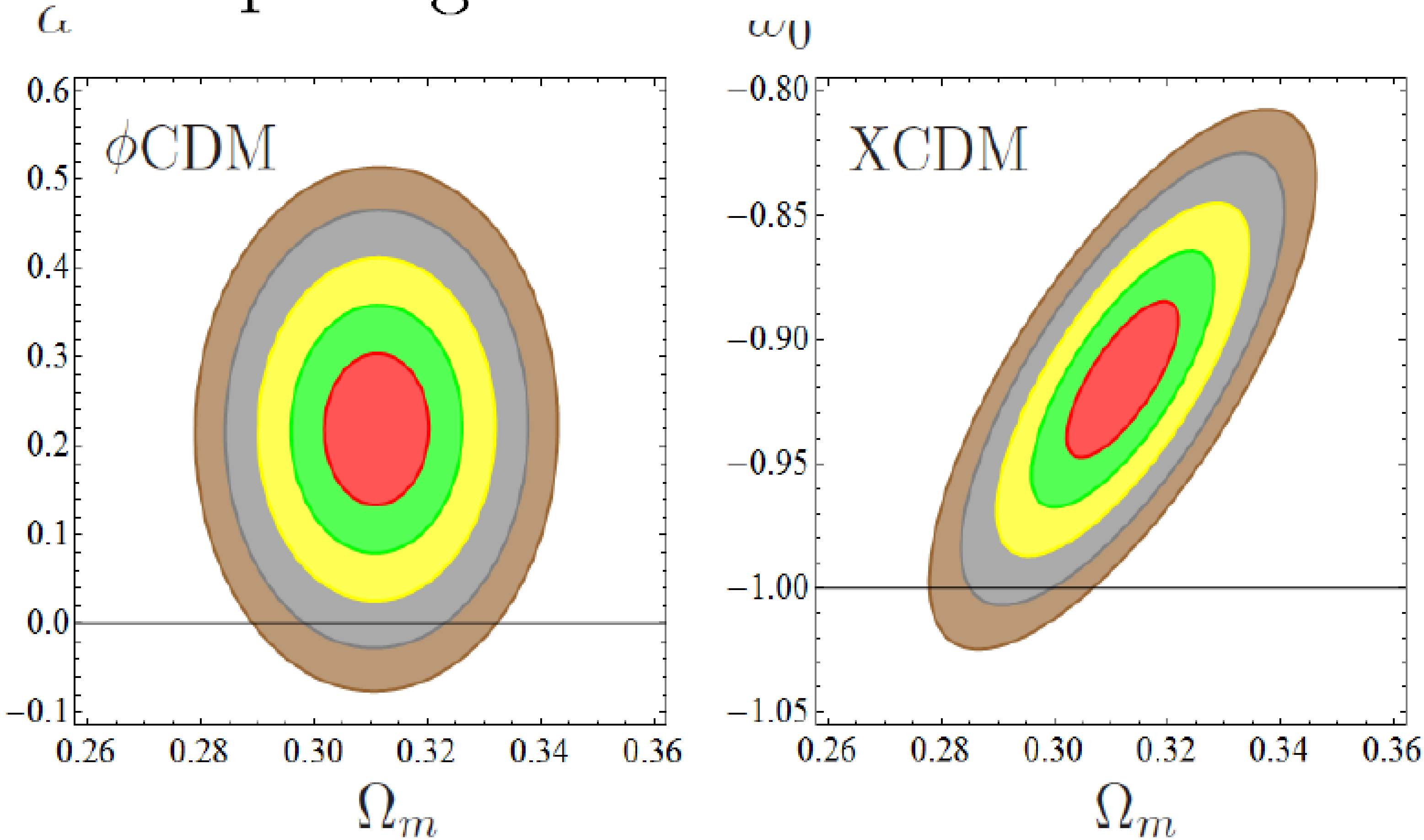




$$h = 0.671 \pm 0.006$$

$$\Omega_m = 0.311 \pm 0.006$$ J 2016)

Comparing ΦCDM and XCDM



$\alpha = 0.219 \pm 0.057$
$h = 0.671 \pm 0.006$
$\Omega_m = 0.311 \pm 0.006$

Summarized conclusions

- **Dynamical DE** could be explained by a **variable Λ** term in interaction with matter or **G**

- **Running vacuum models** seem to describe **better** the observations

  $\mathrm{SNIa+BAO+}H(z)\mathrm{+LSS+CMB}$ **than the ΛCDM**

  at 4σ confidence level !!

- The possible **variation** of the **fundamental constants**
  could be a hint of the vacuum dynamics with matter

- These ideas may signal a **connection** between the
  the **LS structure** and the **quantum phenomena**
  in the **microcosmos**

- Perhaps they can also give some useful clue to solve
  the famous and tough **CC problem** **!!**